Dark Energy and Matter in 4 Dimensions From an Empty Kaluza-Klein Spacetime

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Abstract

We consider the third order Lovelock equations without the cosmological constant term in an empty $n(\geq 8)$-dimensional Kaluza-Klein spacetime $\mathcal{M}^4 \times \mathcal{K}^{n-4}$, where $\mathcal{K}^{n-4}$ is a constant curvature space. We show that the emptiness of the higher-dimensional spacetime imposes a constraint on the metric function(s) of 4-dimensional spacetime $\mathcal{M}^4$. We consider the effects of this constraint equation in the context of black hole physics, and find a black hole solution in 4 dimensions in the absence of matter field and the cosmological constant (dark energy). This solution has the same form as the 4-dimensional solution introduced in [10] for Gauss-Bonnet gravity in the presence of cosmological constant, and therefore the metric of $\mathcal{M}^4$ which satisfies the vacuum Lovelock equations in higher-dimensional Kaluza-Klein spacetime is unique. This black hole solution shows that the curvature of an empty higher-dimensional Kaluza-Klein spacetime creates dark energy and matter with non-traceless energy-momentum tensor in 4 dimensions.

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I. INTRODUCTION

High-precision observational data have confirmed with startling evidence that the universe is undergoing a phase of accelerated expansion [1]. The cause of this acceleration still remains an open and tantalizing question. In the standard cosmological model, where the acceleration of the universe is taken into account by a positive cosmological constant term, dark energy is responsible for the acceleration of the universe (see Ref. [2] for a review and references therein). The value of the energy density stored in the cosmological constant today, has to be of order of the critical density, namely $10^{-3} \ eV^4$. This value seems arbitrarily small and the known mechanisms, such as the popular TeV-scale supersymmetry breaking scenario or any top-down high-scale particle physics mechanisms, fail to produce it.

Rather than dealing directly with the cosmological constant, one may also explore the alternative view point through a modified gravity approach. A very promising way to explain these major problems is to assume that at large scales, or higher dimensional spacetime, Einstein theory of General Relativity breaks down and a more general action should describe the gravitational field. In this context, an interesting possibility is the existence of extra dimensions. It is widely believed that string theory is moving towards a viable quantum gravity theory, and one of the key predictions of string theory is precisely the existence of extra spatial dimensions. Extra dimensional models have proven to be very fruitful in providing new ways of attacking the old problems. In this context, in recent years, theories with large extra dimensions have received an explosion of interests. New models such as brane scenarios [3], large extra dimensions, and warped extra dimensions have not only revolutionized the Kaluza-Klein theory [4], but also shed new light on some long-standing problems in particle physics and cosmology. Interestingly, theories with large extra dimensions can be even tested by future collider experiments [5]. A natural modification of gravity in higher dimensions is Lovelock gravity, whose Lagrangian consist of the dimensionally extended Euler densities. This Lagrangian is obtained by Lovelock as he tried to calculate the most general tensor that satisfies properties of Einstein's tensor in higher dimensions [6]. Since the Lovelock tensor contains derivatives of metrics of order not higher than second, and the second order derivative is linear in the field equations, the initial value problem is well formulated and the evolution of the gravitating system is uniquely defined.

One of the early works of physicists who consider gravity in higher-dimensional space-

time was the attempt of Kaluza and Klein, who split general relativity and electrodynamics from an empty 5-dimensional spacetime [7]. Kaluza unified not only gravity and electromagnetism, but also matter and geometry, for the photon appeared in four dimensions as a manifestation of empty five-dimensional spacetime. Since the theory of gravity with the assumptions of Einstein in higher dimensions is Lovelock gravity, it is natural to use the Lovelock field equations of gravity instead of the Einstein equation in the context of Kaluza-Klein theory. The action of Lovelock gravity may also be viewed as the low energy effective action of string theory [8]. The generation of dark energy from a (super) string effective action with higher order curvature corrections and a dynamical dilaton has been investigated in [9]. Here, we want to consider third order Lovelock gravity without a cosmological constant term in an n-dimensional empty spacetime which is the product of a 4-dimensional Lorentzian manifold $\mathcal{M}^4$ with an $(n-4)$-dimensional constant curvature manifold $\mathcal{K}^{n-4}$. In the context of black hole physics, we investigate the problem of having a 4-dimensional asymptotically anti-de Sitter (AdS) or de Sitter (dS) charged black hole in the absence of electromagnetic field and the cosmological constant term in the field equations of gravity. This idea has been used for the Gauss-Bonnet gravity, but the authors were forced to keep the cosmological constant in higher dimensions which weakens the idea of empty higher-dimensional spacetime [10, 11].

The out line of this work is as follows. In Sec. II, we decompose the field equations of third order Lovelock gravity in an n-dimensional spacetime which is homomorphic to $\mathcal{M}^4 \times \mathcal{K}^{n-4}$, into two equations and introduce a constraint equation for the vacuum solutions of the field equation. In Sec. III, we present a new 4-dimensional asymptotically (A)dS charged black hole solution in the absence of cosmological constant and electromagnetic field. Finally, we give some concluding remarks.

II.   KALUZA-KLEIN DECOMPOSITION OF BASIC EQUATIONS

The vacuum gravitational field equations of third order Lovelock gravity may be written as:

$$\mathcal{G}_{\mu\nu} = G_{\mu\nu}^{(1)} + \alpha_2 G_{\mu\nu}^{(2)} + \alpha_3 G_{\mu\nu}^{(3)} = 0, \tag{1}$$

where α_i's are Lovelock coefficients, $G^{(1)}_{\mu\nu}$ is just the Einstein tensor, and $G^{(2)}_{\mu\nu}$ and $G^{(3)}_{\mu\nu}$ are the second and third order Lovelock tensors given as

$$G^{(2)}_{\mu\nu} = 2(R_{\mu\sigma\kappa\tau}R_\nu{}^{\sigma\kappa\tau} - 2R_{\mu\rho\nu\sigma}R^{\rho\sigma} - 2R_{\mu\sigma}R^\sigma{}_\nu + RR_{\mu\nu}) - \frac{1}{2}\mathcal{L}_2 g_{\mu\nu}, \tag{2}$$

$$\begin{aligned}
G^{(3)}_{\mu\nu} = &-3(4R^{\tau\rho\sigma\kappa}R_{\sigma\kappa\lambda\rho}R^\lambda{}_{\nu\tau\mu} - 8R^{\tau\rho}{}_{\lambda\sigma}R^{\sigma\kappa}{}_{\tau\mu}R^\lambda{}_{\nu\rho\kappa} + 2R_\nu{}^{\tau\sigma\kappa}R_{\sigma\kappa\lambda\rho}R^{\lambda\rho}{}_{\tau\mu} \\
&- R^{\tau\rho\sigma\kappa}R_{\sigma\kappa\tau\rho}R_{\nu\mu} + 8R^\tau{}_{\nu\sigma\rho}R^{\sigma\kappa}{}_{\tau\mu}R^\rho{}_\kappa + 8R^\sigma{}_{\nu\tau\kappa}R^{\tau\rho}{}_{\sigma\mu}R^\kappa{}_\rho \\
&+ 4R_\nu{}^{\tau\sigma\kappa}R_{\sigma\kappa\mu\rho}R^\rho{}_\tau - 4R_\nu{}^{\tau\sigma\kappa}R_{\sigma\kappa\tau\rho}R^\rho{}_\mu + 4R^{\tau\rho\sigma\kappa}R_{\sigma\kappa\tau\mu}R_{\nu\rho} + 2RR_\nu{}^{\kappa\tau\rho}R_{\tau\rho\kappa\mu} \\
&+ 8R^\tau{}_{\nu\mu\rho}R^\rho{}_\sigma R^\sigma{}_\tau - 8R^\sigma{}_{\nu\tau\rho}R^\tau{}_\sigma R^\rho_\mu - 8R^{\tau\rho}{}_{\sigma\mu}R^\sigma{}_\tau R_{\nu\rho} - 4RR^\tau{}_{\nu\mu\rho}R^\rho{}_\tau \\
&+ 4R^{\tau\rho}R_{\rho\tau}R_{\nu\mu} - 8R^\tau{}_\nu R_{\tau\rho}R^\rho{}_\mu + 4RR_{\nu\rho}R^\rho{}_\mu - R^2 R_{\nu\mu}) - \frac{1}{2}\mathcal{L}_3 g_{\mu\nu}. \tag{3}
\end{aligned}$$

In Eqs. (2) and (3) $\mathcal{L}_2 = R_{\mu\nu\gamma\delta}R^{\mu\nu\gamma\delta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ is the Gauss-Bonnet Lagrangian and

$$\begin{aligned}
\mathcal{L}_3 = &2R^{\mu\nu\sigma\kappa}R_{\sigma\kappa\rho\tau}R^{\rho\tau}{}_{\mu\nu} + 8R^{\mu\nu}{}_{\sigma\rho}R^{\sigma\kappa}{}_{\nu\tau}R^{\rho\tau}{}_{\mu\kappa} + 24R^{\mu\nu\sigma\kappa}R_{\sigma\kappa\nu\rho}R^\rho{}_\mu \\
&+ 3RR^{\mu\nu\sigma\kappa}R_{\sigma\kappa\mu\nu} + 24R^{\mu\nu\sigma\kappa}R_{\sigma\mu}R_{\kappa\nu} + 16R^{\mu\nu}R_{\nu\sigma}R^\sigma{}_\mu - 12RR^{\mu\nu}R_{\mu\nu} + R^3 \tag{4}
\end{aligned}$$

is the third order Lovelock Lagrangian. Equation (1) does not contain the derivative of the curvatures, and therefore the derivatives of the metric higher than two do not appear. In order to have the contribution of all the above terms in the field equation, the dimension of the spacetime should be equal or larger than seven. Solutions of third order Lovelock gravity have been introduced in [12]. Here, we investigate the Kaluza-Klein solutions of third order Lovelock gravity.

Consider an n-dimensional spacetime which is homomorphic to $\mathcal{M}^4 \times \mathcal{K}^{n-4}$ for $n \geq 8$, with the metric

$$ds^2 = g_{ab}dx^a dx^b + r_0^2 \gamma_{ij} d\theta^i d\theta^j, \tag{5}$$

where $a, b = 0, 1, 2, 3$ and $i, j = 4...(n-1)$. In Eq. (5) g_{ab} is a Lorentzian metric on $\mathcal{M}^4$, r_0 is a constant and γ_{ij} is the metric on the $(n-4)$-dimensional constant curvature space $\mathcal{K}^{n-4}$ with curvature $(n-4)(n-5)k$, where $k = 0, \pm 1$. It is a matter of calculation to show that the tensor $\mathcal{G}^\mu{}_\nu$ gets the below decomposition

$$\begin{aligned}
\mathcal{G}^a{}_b = &-\frac{(n-4)(n-5)k}{2r_0^2}\left\{1 + \frac{k\alpha_2(n-6)(n-7)}{r_0^2} + \frac{\alpha_3(n-6)(n-7)(n-8)(n-9)}{r_0^4}\right\}\delta^a{}_b \\
&+ \left\{1 + \frac{2k\alpha_2(n-4)(n-5)}{r_0^2} + \frac{3\alpha_3 k^2(n-4)(n-5)(n-6)(n-7)}{r_0^4}\right\}\hat{G}^a{}_b, \tag{6}
\end{aligned}$$

$$\mathcal{G}^i{}_j = -\frac{1}{2}\left\{\frac{(n-5)(n-6)k}{r_0^2}\left[1+\frac{k\alpha_2(n-7)(n-8)}{r_0^2}+\frac{k^2\alpha_3(n-7)(n-8)(n-9)(n-10)}{r_0^4}\right]\right.$$
$$+\left[1+\frac{2k\alpha_2(n-5)(n-6)}{r_0^2}+\frac{3k^2\alpha_3(n-5)(n-6)(n-7)(n-8)}{r_0^4}\right]\hat{R}$$
$$\left.+\left[\alpha_2+\frac{3k\alpha_3(n-5)(n-6)}{r_0^2}\right]\hat{L}_2\right\}\delta^i{}_j \tag{7}$$

where the superscripts "hat" represent quantities on $\mathcal{M}^4$.

Now as in the case of Kaluza-Klein theory, we want to obtain the vacuum solutions of Eqs. (6) and (7). In general, $\hat{G}^a{}_b \neq 0$ (as we will see in the next section), and therefore $\mathcal{G}^a{}_b = 0$ if the two brackets in Eq. (6) vanish, which concludes that $k \neq 0$. For $k = \pm 1$, one obtains:

$$\alpha_2 = -\frac{2k(n-6)(n+1)}{(n-4)(n-5)(n^2-5n-18)}r_0^2, \tag{8}$$

$$\alpha_3 = \frac{(n^2-5n-2)}{(n-4)(n-5)(n-6)(n-7)(n^2-5n-18)}r_0^4. \tag{9}$$

Note that n should be greater than 7 ($n \geq 8$) in order to have a finite value for α_3. It is worthwhile to mention that the Lovelock coefficients α_2 and α_3 are proportional to r_0^2 and r_0^4, respectively, where r_0 is the size of the extra dimensions. Thus, the smallness of r_0 is consistent with the fact that the Lovelock coefficients are supposed to be very small. Substituting the above α_2 and α_3 in Eq. (7), one obtains:

$$r_0^4\hat{L}_2 - kA_n r_0^2\hat{R} + B_n = 0, \tag{10}$$

where

$$A_n = \frac{4(n-2)(n-3)(n-5)(n-7)}{n^3-6n^2+11n-54}, \tag{11}$$

$$B_n = \frac{8(n-5)(n-7)(2n^3-27n^2+97n-54)}{n^3-6n^2+11n-54}. \tag{12}$$

Equation (10) should be used as a constraint on the metric function(s) of the 4-dimensional spacetime $\mathcal{M}^4$.

It is worth noting that one needs two parameters in Eq. (6) in order to have $G^a{}_b = 0$. In Ref. [10], the two parameters have been chosen to be the cosmological constant and Gauss-Bonnet coefficient, while in this paper we choose them to be the second and third order Lovelock coefficients. The presence of the cosmological constant in Ref. [10] weakens the idea of emptiness of higher-dimensional Kaluza-Klein spacetime. Of course, one may

use any order of Lovelock gravity with at least two parameters, but in any order of Lovelock gravity the constraint equation (10) for $\mathcal{M}^4 \times \mathcal{K}^{n-4}$ has the same form, and therefore the 4-dimensional solution which satisfies the vacuum Lovelock equations is unique.

III. FOUR-DIMENSIONAL BLACK HOLE SOLUTIONS

In this section, we investigate the effects of the constraint equation (10) in the context of black hole physics. In order to do this, we use the metric of spherically symmetric spacetime in the Schwarzschild gauge, $g_{tt}g_{rr} = -1$, for $\mathcal{M}^4$ manifold:

$$g_{ab}dx^a dx^b = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Sigma_{2(\hat{k})}^2, \tag{13}$$

where $d\Sigma_{2(\hat{k})}^2$ represents the line element of a 2-dimensional hypersurface with constant curvature $2\hat{k}$ and $\hat{k} = 0, \pm 1$. One can show that Eq. (10) for the above metric after integrating two times reduces to

$$2r_0^4 \left(f(r) - \hat{k}\right)^2 + kA_n r_0^2 r^2 \left(f(r) - \hat{k}\right) + \frac{B_n}{12}r^4 - C_1 r + C_2 = 0, \tag{14}$$

where C_1 and C_2 are two arbitrary constants of integration. Thus, the metric function $f(r)$ may be written as:

$$f(r) = \hat{k} - A_n \frac{r^2}{4r_0^2} \left(k \pm \sqrt{1 - \frac{2B_n}{3A_n^2} + \frac{8C_1}{A_n^2 r^3} - \frac{8C_2}{A_n^2 r^4}}\right), \tag{15}$$

which has the same form as the metric function introduced by Maeda and Dadhich, and therefore it establishes the uniqueness of Maeda-Dadhich black hole solution [10].

The function $f(r)$ is real provided $r \geq r_{\min}$, where $r_{\min}$ is the largest real root of the square root:

$$(3A_n^2 - 2B_n)r_{\min}^4 + 24C_1 r_{\min} - 24C_2 = 0. \tag{16}$$

For negative values of C_2, the above equation has no real root and therefore the metric function is real for all the spacetime. But for positive values of C_2, for which the solution is asymptotically Reissner-Nordstrom black hole as we will see below, one may restrict the spacetime to the region $r \geq r_{\min}$ by the transformation $\rho^2 = r^2 - r_{\min}^2$ (see the Appendix for more details).

In order to study the general structure of this solution, we first look for the curvature singularities. It is easy to show that the Kretschmann scalar $R_{\mu\nu\lambda\kappa}R^{\mu\nu\lambda\kappa}$ diverges at $r = 0$
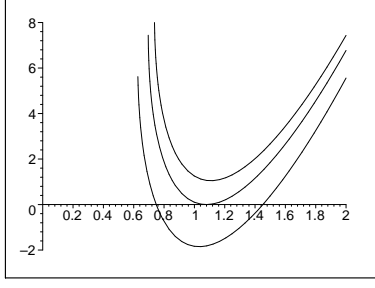
FIG. 1: $f(r)$ versus r for $n = 8$, $k = -1$, $\hat{k} = 1$, $C_2 = 1.0$, $r_0 = .2$, $C_1 < C_{1\text{ext}}$, $C_1 = C_{1\text{ext}}$ and $C_1 > C_{1\text{ext}}$ from up to down, respectively.

or $r = r_{\min}$ for negative or positive values of C_2, respectively. Here, we are interested in black hole solutions, and therefore we only consider the plus branch of Eq. (15). The minus branch presents a naked singularity. Seeking possible black hole solutions, we turn to looking for the existence of horizons. The event horizon(s), if there exists any, is (are) located at the root(s) of $f(r) = 0$:

$$\frac{B_n}{12}r_+^4 - k\hat{k}A_n r_0^2 r_+^2 - C_1 r_+ + C_2 + 2r_0^4 = 0. \tag{17}$$

Equation (17) may have zero, one or two real positive solutions depending on the suitable choices of C_1 and C_2. Thus, the topological solution given by Eqs. (13) and (15) may be interpreted as a naked singularity, an extreme black hole or a black hole with two inner and outer event horizons. For extreme black hole, both $f(r)$ and $f'(r)$ are zero at the horizon $r = r_{\text{ext}}$. Differentiating Eq. (14) with respect to r and using $f(r_{\text{ext}}) = f'(r_{\text{ext}}) = 0$, it is easy to show that:

$$B_n r_{\text{ext}}^3 - 6k\hat{k}r_0^2 A_n r_{\text{ext}} - 3C_1 = 0. \tag{18}$$

Using Eq. (17) and (18) for r_{ext}, one can show that the solution (15) present an extreme black hole provided $C_1 = C_{\text{ext}}$, has two inner and outer horizons provided $C_1 > C_{\text{ext}}$, and a naked singularity if $C_1 < C_{1\text{ext}}$, where $C_{1\text{ext}}$ is

$$C_{1\text{ext}} = \left(\sqrt{A_n^2 r_0^4 + B_n(C_2 + 2r_0^4)} - 2k\hat{k}r_0^2 A_n\right)\left(\frac{8k\hat{k}r_0^2 A_n + 8\sqrt{A_n^2 r_0^4 + B_n(C_2 + 2r_0^4)}}{9B_n}\right)^{1/2}. \tag{19}$$

Figure 1 shows the function $f(r)$ versus r for different values of C_1.

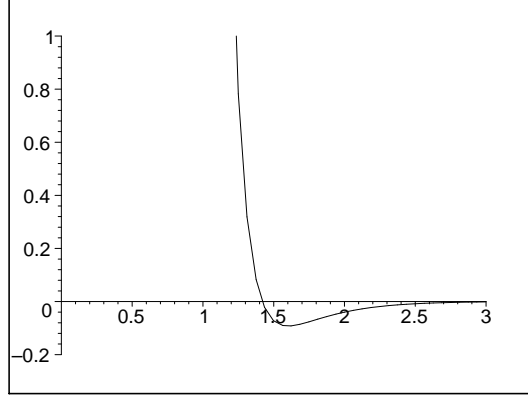It is a matter of calculation to show that the asymptotic behavior of the solution is (A)dS

FIG. 2: Trace of energy-momentum of matter versus r for $n = 8$, $k = -1$, $\hat{k} = 1$, $C_1 = 0.5$, $C_2 = .6$, $r_0 = .2$.

with the effective cosmological constant

$$\Lambda_{\text{eff}} = \frac{3A_n}{4r_0^2}\left(k + \sqrt{1 - \frac{2B_n}{3A_n^2}}\right). \tag{20}$$

Thus, the asymptotic behavior of the solution is dS for $k = 1$, and is AdS for $k = -1$. The metric function $f(r)$ at large value of r may be written as

$$f(r) = \hat{k} - \frac{\Lambda_{\text{eff}}}{3}r^2 - \frac{2m}{r} + \frac{q^2}{r^2}, \tag{21}$$

where

$$m = \frac{C_1}{2r_0^2}\sqrt{1 - \frac{2B_n}{3A_n^2}}, \tag{22}$$

$$q^2 = \frac{C_2}{r_0^2}\sqrt{1 - \frac{2B_n}{3A_n^2}}. \tag{23}$$

Thus, the above solution at large values of r behaves as the Reissner-Nordstorm-(A)dS black hole in the absence of cosmological constant and any kind of electromagnetic field provided $C_2 > 0$. The parameters C_1 and C_2 may be related to the mass and charge parameters of the spacetime according to Eqs. (22) and (23).

In order to investigate the nature of the matter field created by the curvature of the higher-dimensional Kaluza-Klein spacetime in 4 dimensions, we first set the arbitrary integration constants C_1 and C_2 equal to zero. In the absence of matter ($C_1 = C_2 = 0$), the 4-dimensional Einstein tensor $\hat{G}_{ab}$ will be proportional to g_{ab}, where the constant of proportionality is the effective cosmological constant. This fact persuades us to split the effective 4-dimensional

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energy-momentum tensor created by the curvature of the higher-dimensional Kaluza-Klein spacetime into a part which is due to the effective cosmological constant and the reminder of it as follows:

$$T_{ab}^{\mathrm{DE}} = -\frac{\Lambda_{\mathrm{eff}}}{8\pi}g_{ab}, \tag{24}$$

$$T_{ab}^{\mathrm{M}} = \frac{1}{8\pi}\left(\hat{G}_{ab} + \Lambda_{\mathrm{eff}}g_{ab}\right), \tag{25}$$

where the superscripts "DE" and "M" used for the abbreviation of dark energy and matter, respectively. T_{ab}^{DE} is the energy-momentum tensor of the effective cosmological constant, and T_{ab}^{M} is the energy-momentum tensor of the matter created by the curvature of the higher-dimensional Kaluza-Klein spacetime. The word "dark" is used to emphasize that the origin of dark energy is the curvature of higher dimensional Kaluza-Klein spacetime in Lovelock gravity. Note that at infinity, the effect of matter is vanished, and we leave with the energy-momentum of the dark energy. It is a matter of calculations to show that the radial and tangential pressures of the matter part at large r reduce to

$$P_r^M = -\frac{C_2}{8\pi A_n r_0^2}\left(1 - \frac{2B_n}{3A_n}\right)^{-1/2}\frac{1}{r^4} - \frac{6C_1^2}{8\pi A_n^3 r_0^2}\left(1 - \frac{2B_n}{3A_n}\right)^{-3/2}\frac{1}{r^6} + \mathcal{O}\left(\frac{1}{r^8}\right), \tag{26}$$

$$P_t^M = \frac{C_2}{8\pi A_n r_0^2}\left(1 - \frac{2B_n}{3A_n}\right)^{-1/2}\frac{1}{r^4} + \frac{12C_1^2}{8\pi A_n^3 r_0^2}\left(1 - \frac{2B_n}{3A_n}\right)^{-3/2}\frac{1}{r^6} + \mathcal{O}\left(\frac{1}{r^8}\right), \tag{27}$$

respectively, which are sufficiently small at large r. Also, it is worth to note that $T_t^{t(\mathrm{M})} = T_r^{r(\mathrm{M})}$ and $T_\theta^{\theta(\mathrm{M})} = T_\varphi^{\varphi(\mathrm{M})}$, but the energy momentum tensor T_{ab}^{M} is not traceless at finite r. This can be seen from Eq. (10) which shows that $\hat{R}$ is a linear combination of $\hat{L}_2$ and B_n and therefore is a function of r, that reduces to a constant as r goes to infinity. Figure 2 shows the trace of energy-momentum of matter versus r, which shows that it becomes traceless at infinity. Thus, the electromagnetic field which can be related to this solution is not Maxwellian.

The solution presented in this subsection shows the creation of dark energy and matter out of pure curvature of a higher-dimensional Kaluza-Klein spacetime.

IV. CLOSING REMARKS

We decomposed the field equations of third order Lovelock gravity in the absence of cosmological constant and matter field in an n-dimensional Kaluza-Klein spacetime, $\mathcal{M}^4 \times$

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$\mathcal{K}^{n-4}$, into two equations. One of these equations fixes the Lovelock coefficients in terms of the size of the extra dimensions r_0 and the dimension of the Kaluza-Klein spacetime, and the second one should be used as a constraint on the four dimensional metric of $\mathcal{M}^4$. The proportionality of the Lovelock coefficients and the powers of r_0 [relations (8) and (9)] shows that the smallness of r_0, which is needed for completing the scenario of Kaluza-Klein, is consistent with the smallness of Lovelock coupling constant.

We performed our formalism in the context of black hole physics, and found an asymptotically (A)dS charged black hole solution in four dimensions which behaves like Reissner-Nordtrom-(A)dS black hole at large distance from the singularity of the spacetime. In this scenario, we found that the curvature of higher-dimensional Kaluza-Klein spacetime creates the effects of matter and dark energy in four dimensions. That is, one may have asymptotically (A)dS charged black hole solutions in the absence of the cosmological constant and electromagnetic field. We also found that the electromagnetic field is not Maxwellian. This solution is the same as the solution which is introduced by Maeda and Dadhich, and therefore it establishes the uniqueness of Maeda-Dadhich black hole solution [10].

Now, one may ask about the sources of q and Λ_{eff}. Indeed, the Ricci scalar in an empty 4-dimensional spacetime should be zero, while we found that $\hat{R}$ does not vanish, and depends on the coordinate r. At large r, both $\hat{R}$ and $\hat{L}_2$ become constant and therefore the source of q is asymptotically Maxwellian, while at finite r, the matter created by the curvature of higher-dimensional Kaluza-Klein spacetime is not traceless. It is worthwhile to mention that this creation of dark energy and matter out of pure curvature of a higher-dimensional Kaluza-Klein spacetime is the generalization of the creation of matter out of pure curvature discussed in Ref. [10].

APPENDIX

In order to restrict the spacetime to the physical region $r \geq r_{\text{min}}$, we introduce a new radial coordinate ρ as:

$$\rho^2 = r^2 - r_{\text{min}}^2 \Rightarrow dr^2 = \frac{\rho^2}{\rho^2 + r_{\text{min}}^2} d\rho^2,$$

where now ρ is in the range $0 \leq \rho < \infty$. With this new coordinate, the metric (13) becomes:

$$g_{ab}dx^a dx^b = -f(\rho)dt^2 + \frac{\rho^2 d\rho^2}{(\rho^2 + r_{\min}^2)f(\rho)} + (\rho^2 + r_{\min}^2)d\Sigma_{2(\hat{k})}^2,$$

where now $f(\rho)$ is

$$f(\rho) = \hat{k} - A_n \frac{\rho^2 + r_{\min}^2}{4r_0^2} \left(k \pm \sqrt{1 - \frac{2B_n}{3A_n^2} + \frac{8C_1}{A_n^2(\rho^2 + r_{\min}^2)^{3/2}} - \frac{8C_2}{A_n^2(\rho^2 + r_{\min}^2)^2}} \right).$$

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